EUCLIDES ELEMENTS;

The whole Fifteen Books compendiously Demonstrated

By Mr. ISAAC BARROW Fellow of *Trinity Colledge* in CAMBRIDGE.

And Translated out of the Latine.

~~Where unto is Added~~ *An Introduction to the Art of* Species By *Jonas Moore.*

LONDON,

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A ſhort Introduction

Into the ART

OF

SPECIES.

Uſefull for every one that deſire to be

GEOMETRICIANS.

Fruſtra fit per plura, quod fiat per pauciora.

By *Jonas Moore Profeſſor* of the *Mathemat.*

LONDON, Printed and ſold by *Joſeph Moxon*: at his Shop on *Corn-hill*, at the Signe of *Atlas*. 1660.

An Introduction

To the Art of

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THis peece of the 15 Books of *Euclide*, having been some years since reduced into this portable Form, by the Author, the most Ingenious and painfull *Mathematician*, Mr *Barrow*, (my Honoured Friend,) in the Latine Tongue; I have ever since the same Publication read the same to such Gentlemen as I had the happiness to enter or guide into the Studies *Mathematical*; as being the Basis and Foundation of *Geometry*: However, I found it necessary to premise and teach them some Rules and Principles of the *Art* of *Symbols* or *Species*; and to acquaint them with the meaning and use of this *Analitical* and *Symbolical* way: For as Mr *Oughtred* in the *Preface* to his *Clavis* observes, that very many that have any way used themselves to words, rather then to express the thing by their *Species*, have been much affrighted at this manner of delivery, yet certainly after a little initiation have fallen into great love and admiration with it, as easing the Fancy and Memory, by bringing that suddenly to the mind, which otherwise in Words many Lines, nay sometimes Leaves, would not express.

And there seems to me some necessity of this smal Introduction: First that the Ingenious reader may in the perusal of the book alter, change, and express the Propositions and Demonstrations, so as may be most for his pleasure in this short way: and secondly, because till the First 6 Books of *Euclide* be well understood, it is in vain to read or meddle with Mr *Oughtreds Clavis Mathematicæ*, and yet it will not be amiss to teach some

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few Principles that the whole ſpecifical Doctrine may dayly grow and increaſe.

Mr. *I. H.* Having ſhewed ſome of my notes to this purpoſe to Mr. *Moxon*, (who hath bought a conſiderable part of this Engliſh Impreſſion) I was earneſtly intreated by him that he might publiſh this ſmal tract; that ſuch as were deſirous might have the ſame bound up with the *Euclide*: and though I was very unwilling, not knowing what offence it might give to the worthy Author (being well able to have amended what I had done) yet in his abſence, I have conſented to publiſh theſe *Principia*, being deduced out of the *Clavis Mathematicæ*.

Of Notation.

1. How great the advantage of the Change of letters to theſe numerical Characters long ſince received by us from the *Arabians*, and by them from the *Indians*, was; Time may by degrees undervalue: but the conſideration of thoſe troubleſome Poſitions and tedious opperations of any one that will but try that way, will be a ſufficient motive to the eſtimation; ſome years hence, when *Geometry* ſhall ſo Compendiouſly be handled, and *Lines*, *Angles*, *Figures*, and *Bodies*, be delivered by ſuch Rules and Characters, (that as Mr *Oughtred* ſaith in his *Preface* to his *Clavis*, a *Theorem* may be ſo expreſſed, not onely intelligible to one Nation, but to all the Nations on the Earth, ſuppoſing a few notes to be known.) Then will *Viet* and our worthy *Oughtred* be admired, for laying the groundwork of ſo compleat a Building.

For as all the Learned world do by theſe Characters 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. underſtand whatſoever ſhall be needfull to be expreſſed in number; ſo in time ſhall we in all Nations underſtand one another by Characters in *Geometry*, and more then ſo, ſuch Characters ſhall receive their *Analitic* laws and reſolutions, as may in ſome part appear in this following diſcourſe.

2. The *Notes* or *Characters* uſed in this *Symbolical* way are ſomtimes the Letters of the *Alphabet* A, B, C, D. &c. and ſomtimes Characters to expreſs Names or Terms of Art, as it were after the manner of thoſe that write Short-hand.

3. T

3. The Letters of the *Alphabet* may signifie either Lines, Angles, Figures, *&c.* As for Example, A Line may be noted by two letters set down at the ends of the lines, as here A B, or els by one letter, as at the lower line by the letter A; and nevertheless in another Proposition, those letters may signify any other Line or Figure: and if the line A B or A be divided into five parts, and 2 parts be to be expressed, then are they expressed thus $\frac{2}{5}$ A B or $\frac{2}{5}$ A: but this Section will be made plainer hereafter.

A ——— B

A

4. The Characters used by Mr *Barrow*, are explained before the first page, as = for equal, or the signe of equality, + for more, or to be added, — less, or to be substracted; × for drawing one side into another, as of a square, Rectangle *&c.* √ for the root or side of a square Cube, *&c.* Q or q, for square; C or c, for Cube; and I would advise farther, that these Characters should be taken notice of, ∠ for an acute angle; L for a right angle; ∠ for an obtuse angle; and the ordinary Forms of Triangles, of a Circle, of a Semi-circle, Segment, Sector, of a Square, Oblong, Rombus, Romboydes, to represent those Figures themselves; and because two Parallel, lines of one length signifieth equality thus =, let = be greater, and = lesser; and let two equal lines set upright signify Parallel, thus ||. I have drawn the first six Books of *Euclide*, both Propositions and Demonstrations, after this manner, into one sheet of Paper, to be understood in any Nation, by those that shall onely know a few Characters, and those likewise signifying the thing treated of.

Addition.

Addition joyneth together the *Species* of the same kind. That is, if there be in the *Symbols* to be added more then one of the same kind, add them: or if there be some affirmed, and some denied, take the difference; so there may be a Compendium of the whole.

Example, if A + 2 B + 3 A + 3 D + 11 B + 8 D The addition of the whole, by summing up the *Species* of the same kind, *viz.* A + 3 A is 4 A and 2 B + 11 B is 13 B and 3 D + 8 D is 11 D, so the sum will be 4 A + 13 B + 11 D.

Again, let 5 A + 3 B — 3 A — 16 B be to be added, first 5 A — less by 3 A would be 2 A and 3 B — 16 B would be — 13 B, the sum would be 2 A — 13 B, the same work is to be observed in the Example following.

In Lines.

AB+3 CD—5 AB	a+ 3d+ 15f	A+15D—A
2CD+ 4AB	—13a+ 5d	— 15 D
sum 5 CD	—12a+8d+15f	o

In Angles.

∠acb + 2∠bcd	∠A + ∠B	∠d + ∠f
—2∠bcd+∠dcf	— ∠B	—∠d — ∠f
sum ∠acb +∠dcf	∠ A	o

In Superficies, and Sollids.

ABq + CDq — 2 ABq	Aq + 3 Ac + B
+ 3 CDq + FEq	— 4 B + 5 Aq
sum —ABq+4CDq+FEq	6Aq +3 Ac — 3 B

And note, that care must be taken, not to mingle *Species* of unlike kinds, as Lines with angles, or Superficies with either, or Squares with Cubes, as in the last Examples, where Aq and 3 Ac are not mingled because of divers kinds.

Substraction.

In *Substraction* you are to observe 3 things, 1. The *Species* out of which, 2 The *Species* proposed to be Substracted, 3. The residue, or difference, and then the rule will be. Change all the Signes of the *Species* to be Substracted, that is + to be —, and — to be +, after which work, sum up all as in *Addition*. As in this Example, out of 3 A — 2 B take 5 D — 2 B. first I set them down, changing the signes of the latter, Thus, 3 A — 2 B — 5 D + 2 B, then the sum is 3 A — 5 D, which

is

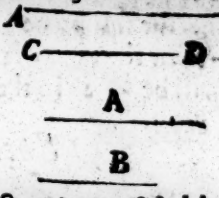
is the difference: and for proof, if you add 3 A — 5 D the remainder to 5 D — 2 B, the sum will be 3 A — 2 B the *Species ex quo*.

5 A + 3 D + 15 C	∠A + ∠B
— 9 A + 15 C	+ ∠B
5A + 3D + 15C + 9A — 15C	∠A + ∠B — ∠B
diff. 14 A + 3 D	diff. ∠A.

A + B	Aq + 2Æ + Bq
A — B	+ 2Æ — Bq
A + B — A + B	Aq + 2Æ + Bq — 2Æ + Bq
diff. 2 B.	diff. Aq + 2Bq.

Multiplication.

Multiplication or drawing one *Species* or Line into another, is to joyn them together with this signe X, if they consist of two letters: or without any signe, if of one letter: as if the two lines *A B* and *C D* were to be *Multiplied*, or to be drawn into a Rectangled *Paralellogram*, it would be *A B* X *C D*, but if the same lines were only noted by two letters *A* and *B*, then the Plain or Rectangle or Multiplication would be *A B* or *B A*, for if single letters be joyned together without any note at all, then are they Multiplied or drawn together; as if *C B* stand together and signifie two lines, they are understood to be the Plain or Rectangle of the two lines C and *B*.



Multiplication of single Species.

AB	*AB*	*ABq*	2*CD*	2*FG*
BC	*AB*	*AB*	3*CD*	5*CD*
*AB*X*BC*	*ABq*	*ABc*	6*CDq*	10 *FG*X*CD*

The last 5 Examples are of *Species* marked with two letters, the Multiplication of the first Example makes the Plain and Rectangle of A B X B C, or a Rectangle whose sides shall be A B and B C. The second Example is the drawing of the

side

ſide A B, or line A B into it ſelf, which makes a Square or a Rectangle whoſe ſides are equal, and is marked with q for a Square; for it is all one to expreſs A B × A B, or to ſay A B q. The third Example draws the Square A B q into the ſide A B which makes a Cube; if 3 Dimenſions for the Plain or Square A B × A B of two, being drawn into A B makes A B × A B × A B = A B q × A B = A B c, the Cube being expreſſed by c. In the two laſt Examples where numbers are joyned with *Species*; there is no other difference, only in Multiplying the numbers together, and prefixing them before the *Species*: and if the lines be noted with one letter, the ſame Examples are thus expreſſed.

A	A	*A*q	2D	2 F
B	*A*	*A*	3D	5 D
pl. *B* A	*A*q	*A*c	6 Dq	10 D F

But if the notes be unlike, and there be more *Symbols* then one to be multiplied, then conſideration muſt be had of the ſigns: for if they be both of one kind; that is, both + or both — then the note of the plane will be +, but if they be unlike it will be — as in the Examples.

Species by two Letters.

—*A B*	+*A B*	*A B* + C D — C F
—*B* C	—*B* C	+ *A B*
pl. +*BA*×*BC*	—2*AB*×*BC*	*AB*q + *AB*×CD — AB×*CF*

Species by one Letter.

A + B	*A* + *B*	*A* + B	*A* — B
A	*B*	*A* + B	*A* — B
*A*q + *BA*	*BA* + Bq	*A*q + B*A*	*A*q — B*A*
		+ B*A* + Bq	— B*A* + Bq
		*A*q + 2B*A* + Bq	*A*q — 2B*A* + Bq

Diviſion, or Application.

As *Multiplication* or drawing one ſide or line into another makes

makes up the Rectangle, so *Division* or *Adplication* dissolves the same, and findeth out the one side by the other: which being considered, the rule will be onely this, *Observe what* Species *being Multiplied by the Divisor will produce the Dividend, and set it down for the Quotient*, and note that like signs make more, and unlike less, as in *Multiplication*.

Suppose you were to divide $AB \times CD$ by CD. the Quotient would be AB for $AB \times CD =$ to the Dividend.

$CD)AB \times CD(AB$

Examples of single Letters.

$$\begin{array}{l} D^{r}\quad D^{d}\qquad\qquad Q^{t} \\ A)BA+CA(B+C \end{array} \qquad a)aq+2ae+aeq(a+2e+eq$$

$$\begin{array}{l} A+E)\ Aq+2AE+Eq\ (A+E \\ \qquad\quad Aq+AE \\ \hline \qquad\quad AE+Eq \\ \qquad\quad AE+Eq \\ \hline \qquad\qquad\quad 0 \end{array} \qquad B)Bq\ (B$$

But if such an *Application* or *Division* cannot be made as may easily be seen by the *Composition* of the *Dividend*, then must you set the *Divisor* over the *Dividend* in form of a *Fraction*: as if you were to *Divide* B by A, or apply A to B, it would be $\frac{B}{A}$ so $C+2D$ divided by $A+F$ would be $\frac{A+F}{C+2D}$.

Of Equation.

An Equation we call that, when some *Species* of Lines, or Angles, or Superficies, or Solids, *&c.* being of the same kind, are equal to other *Species* of Lines, Angles, Superficies or Solids, *&c.* of the same kind: and are expressed in *Species* thus, $B+C=D$. $\angle D = \angle A + \angle D$. $BA = CA$. $Aq = CA$. $AC = DC$ to be read thus: The line B added to the line C are equal to the line D. The angle D is equal to the angle A added to the angle D. The plane of B into A is equal to the plane of C into A, The square of A is equal to the

the plane of C in *A*. The Cube of *A* = to the Cube of D, so that where the sign of Equality is, there is imagined some *Species* on each side equal, as *A* = B.

And for expressing the *Axioms* in *Euclide*, and addition of some few more, take these that follow.

1. If *A* = B and B = C, then *A* = B = C = *A*.

By Addition.

2. If *A* = B then *A* + D = B + D. If *A* − D = B add D it will be *A* = B + D.

Substraction.

3. If A = B substract C from either side of the Equation it will be *A* − C = B − C, or if A + D = B substract D it will be A = B − D.

By Multiplication,

4 If both sides of the Equation be multiplied by the same or by equal *Species*, the Equation will still be true. If A = B multiply both by D it will be D A = B D, and $\frac{A}{D} = \frac{B}{D}$ multiply both by D it will be A = B D, and if $\frac{D}{BC} + \frac{A}{BC} = F$ it will be D + A = F B *C* by multiplying all by B *C*, If A = B and *C* = D it will be *C* A = B D.

By Division.

5. If both sides of the Equation be divided by the same or equal *Species*, the Equation will be true.

If A = B divide both by D it will be $\frac{A}{D} = \frac{B}{D}$, If D A = B *C* divide all by D, it will be A = $\frac{B\,C}{D}$

6. If 2 A = D and D = 2 B then 2 A = 2 B.

7. If B be $\frac{1}{2}$ of A and D be $\frac{1}{2}$ of A then B = D = $\frac{1}{2}$A,

15. If A = B and D = *C* then A + D less by B + *C* = D − *C*

16. If

16. If D = C and A = B, then A + D less by B + C = D — C
17. If A = B and D = C, then A — D less by B — C = D — C
18. If D = C and A = B, then A — D less by B — C = D — C
19. If Z a line &c. be divided into 3 parts A, B, C, then Z = A + B + C.
20. If Z = 2 A take D from the Equation, it will be Z — D = 2 A — D.

All which *Axioms* may for more plainenes to a young Studient be expressed in numbers.

Of the First Book of Euclide.

THe first *Proposition* of *Euclide*, is a *Probleme*, or somthing to be done: and note that the words at the beginning is the *Proposition* in a black letter, and omitting the letters (A B) and A C B) is general, *As upon a finite right line given, to describe an Equilateral Triangle*, the which is to be understood of all the *Propositions* following; but to avoid the repetition of the *Proposition* again with the Figure, the letters belonging to the Figure are so placed in the *Proposition*, as at once both the *Proposition* and *Figure* are explained together.

In every *Proposition* generally as in this, there is some lines or Circles to be drawn or done in order to the performance of it, if it be a *Probleme*: or in order to the Demonstration of it, if it be a *Theorem*: which here is comprehended under these words at large: *From the Centers A and B at the distance of the lines* A B *or* B A *discribe two Circles (by the third postulate or petition, which is upon any Center or at any distance to discribe a Circle)* A F *and* D B D *cutting each other in the point C; from which point C draw (by the first petition, which is, from any point to any point to draw a right line) two right lines C* A *and C* B. Thus much comprehends the preparation or work, and together with the assertion, which is here, and in like cases understood, viz. *(I say the Triangle* A B C *is an Equilateral Triangle)* proceeds the demonstration which follows. *Because the line* A C *is equal to the line* A B *by the* 15 *Definition* (which is, *That all lines from the Center to the Circumference of a Circle are equal) they passing from the Center* A *to the Circumference at C and* B: *and the line* A B *is equal to the line* B C *by the same reason both being drawn from the Center C to the Circumference* A;

and

and C. *Now having proved that both* AC *and* CB *are equal to* AB *by the first* Axiome, which is (*That things equal to the same third, are also equal one to another, It followeth that all the three lines* AC, AB, BC *are equal*: lastly followeth the Conclusion, *wherefore according to the* 23 *Definition* (which is *That an Equilateral Triangle hath three equal sides*) *The Triangle* A C B *is an equilateral Triangle, which was to be done*: All which how briefly and plainly they are expressed by the letters and notes in the Margent do most evidently appear.

The second and third Propositions are Problemes, and demonstrated after the former manner.

The fourth is a Theorem, and doth not only depend on, but indeed is the very 8 *Axiome*; although it be here demonstrated by the 14. *Axiome*: for if the lines BC and EF should not agree, they would include or contain a space contrary to that *Axiome*.

When the young Student comes at the 5th Proposition let him in a by paper set down severally the Triangles there named. First the Triangles A C D and A B E apart, and consider them after the manner of the 4th Proposition: and secondly the Triangles B C E and B C D, and the demonstration will more easily be understood.

The sixth Proposition is not easily to be passed over, but seriously to be regarded, as being the first of that kind that by a deduction to an absurdity or an impossibily demonstrates the thing to be so, because it is impossible to be otherwise: for whersoever you shall take the point D, the Conclusion will be the same.

The 15 Proposition agrees to the 3d *Axiome*, for if the $\angle AEC + \angle CEB = \angle AEC + \angle AED$, substract $\angle AEC$ from both sides, it leaveth $\angle CEB = \angle AED$, And thus much of the first Book.

Of the Second Book of Euclide.

LEt Z be the whole line, A the greater segment, and E the lesser; so that $Z = A + E$ and let $A - E = X$, the which may likewise be expressed in numbers putting $Z = 12$. $A = 9$ $E = 3$ $X = 6$ and let B another line or number be 5. Then

Prop. 1.

If $Z = A + E$ multiply all by B, it will be $ZB = BA +$ BE

B E by the 4 *Axiome* before, which in words is the first Proposition.

Prop. 2.

If $Z = A + E$ multiply all by Z it will be $Zq = ZA + ZE$ per 4 *Axiome* which is the 2 Proposition.

Prop. 3.

If $Z = A + E$ multiply all by A or E it will be $ZA = Aq + E$ or $ZE = AE + Eq$ per 4 *Axiome*.

Prop. 4.

If $Z = A + E$ multiply both sides by themselves which are equal $Z \times Z = Zq$ and $A + E$ into $A + E = Aq + 2AE + Eq$ it will be $Zq = Aq + 2E + Eq$ per 4 *Axiome*.

Prop. 5.

Let $\frac{Z}{2} = \frac{A}{2} + \frac{E}{2}$ which is half the line and $\frac{A}{2} + \frac{E}{2} - E$ represent the Intersegment, multiply these two severally by themselves.

$$\frac{A}{2} + \frac{E}{2} \times \frac{A}{2} + \frac{E}{2}$$

$$\frac{Aq}{4} + \frac{Æ}{4}$$

$$+ \frac{Æ}{4} + \frac{Eq}{4}$$

$$\frac{Aq}{4} + 2\frac{Æ}{4} + \frac{Eq}{4}$$

$$\frac{A}{2} + \frac{E}{2} - E$$

$$\frac{A}{2} + \frac{E}{2} - E$$

$$Aq + \frac{Æ}{4} - \frac{Æ}{2}$$

$$+ \frac{Æ}{4} + \frac{Eq}{4} + \frac{Eq}{4}$$

$$- \frac{Æ}{2} - \frac{Eq}{2} - Eq$$

$$\frac{Aq}{4} - 2\frac{Æ}{4} + \frac{Eq}{4}$$

The

The product $\frac{Aq}{4} + \frac{2Æ}{4} + \frac{Eq}{4}$ and $\frac{Aq}{4} - \frac{2Æ}{4} + \frac{Eq}{4}$ differ only by $Æ$ wherefore add AE to $\frac{Aq}{4} - \frac{2Æ}{4} + \frac{Eq}{4}$ it will make the Equation, that is, $Q\frac{Z}{2} = A\frac{E}{4} + Q\frac{A}{2} + \frac{E}{2} - E = Q$ of the Inter-segment C D.

I shall add no more; having already exceeded my limitation.

Jonas Moore.